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A MODEL FOR STATISTICAL ESTIMATION OF EDUCATIONAL PROCESS QUALITY

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Abstract: *A mathematical model for estimation of the educational process quality through realizing a competent approach is offered in this article. The basis for the model is the theoretic-probabilistic formalization of the process of competence formation. The educational process is seen as the reflection of time and material resources allocated into obtaining competence.*

Keywords*: educational process, competence, quality, probability, function of distributions.*

1. Introduction

At present requirements have become stricter to competitiveness, professional mobility and qualification level of graduates on the labor market. These qualities are determined by their competitiveness which is estimated in its turn by the educational process quality. Under «the educational process quality» we understand the coherence of educational process of the higher educational institution and its graduates to the demands of all the parties concerned (government, applicants for entry, students, graduates, staff and employers). The quality is ensured through realizing opportunities determined by the resources available for reaching this goal.

Nowadays the educational process quality is generally judged through assessing an average level of graduates' competence.

Taking into consideration that due to matter of chance such assessment can not be quite as it is influenced by a number of different factors besides educational process quality, it is necessary to have a set of tools that will allow consideration accidental character of its results. A stochastic model for estimating the level of competencies achieved at different stages of educational process can serve as such set.

The main goal of this article is to work out such model.

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2. Main text

Formally the level of competences achieved during the educational process can be presented by a matrix

$$X = \|x_{ik}\|,\ (0 \le x_{ik} \le 1),\ i = 1,2,\dots,I,\ k = 1,2,\dots,K\ , \qquad (1)$$

where x_{ik} - i- competence level achieved by a quantity of students.

I – quantity of competences formed by an educational institution when realizing the examined stage of the educational process;

K - quantity of students participating in the process.

Matrix (1) components are determined in the first approximation by the ratio:

$$x_{ik} = \frac{\sum_{n \in N_i} d_{nk}}{\sum_{n \in N_i} d_n},\ i = 1,2,\dots,I,\ k = 1,2,\dots,K, \qquad (2)$$

where n- index of the subject;

N_i- a set of subjects forming i-competence;

d_{nk}- a set of points received by the students in a subject;

d_n- a maximum quantity of points that can be received in a subject.

To reach the goals of an educational process is formally to ensure the established level of competences (1).

Hence, formally the goal can be presented as

$$Ц = \{x_{1k} \ge x_1^{д}, x_{2k} \ge x_2^{д}, \dots, x_{Ik} \ge x_I^{д}\},\ k=1,2,\dots,K\ , \qquad (3)$$

where $x_i^{д}$ (i=1,2,…,I) - minimum admissible levels of students' competences for the parties concerned.

Taking into account the adopted notations the educational process can be presented as mapping Q of available material and time resources in a set of graduates' competences i.e.

$$Q(Z\ \ ) \Rightarrow X\ \ , \qquad (4)$$

where Z - allocated resources for realization of the examined stage of an educational process.

If in addition to that

$$X \in Ц\ , \qquad (5)$$

then the results of the examined educational process satisfy the requirements; if condition (5) is not complied with the goals of the process will not be reached.

At the established distribution of material and time resources the results of the educational process are generally non-determined. It is caused by a number of both objective and subjective factors and can manifest itself in the differences of the levels of competences received by the students when realizing the examined stage of an educational process (differences of row elements in matrix (1)).

Non-determination of the educational process results can formally be considered by examining competence matrix X as statistical population sample of distribution of a random quantity of the formed competence levels.

In view of the accidental nature of the formed competence levels during the educational process, reaching its goal is characterized by the occurrence of a random event

$$\hat{X} \in Ц \,. \tag{6}$$

Hence, educational process quality estimation lies in determining probability $P(\hat{X} \in Ц)$ of the fact that the results of an educational process ensure advance of the defined goals, i.e. quality level S equals:

$$S = P(\hat{X} \in Ц). \tag{7}$$

In the general case determination of S quantity is connected with essential analytical difficulties. They are conditioned by the fact that the probability of event (6) is thought of as a joint probability of occurrence I of interconnected events : $\hat{x}_i \geq x_i^{д}$, (i=1,2,…,I), generated by the educational process. The task is a little simplified if we assume that events $\hat{x}_i \geq x_i^{д}$, (i=1,2,…,I) are independent. On such assumption educational process quality estimation takes the form :

$$\bar{S} = \prod_{i=1}^{I} P_i(\hat{x}_i \geq x_i^{д}), \tag{8}$$

where $P_i(\hat{x}_i \geq x_i^{д})$– probability of event occurrence $\hat{x}_i \geq x_i^{д}$.

As $\bar{S} \leq S$ this ratio (8) characterizes the lower bound of quality index of the examined educational process and, hence, adopted assumption of independence of competences intensifies the quality estimation.

To determine the probabilities that appear in the ratio (8) , it is necessary to know their distribution functions. As random quantity $\hat{x}_i$, $(i = 1,2,\dots,I)$ of competence levels are continuous and can range in values $(0 \leq x_i \leq 1)$ it is expedient to plot their distribution functions in the group of beta-functions. Beta distribution is characterized by the density of probabilities (1)

$$f(x_i) = \begin{cases} \frac{\Gamma(\alpha_i+\beta_i)}{\Gamma(\alpha_i)\Gamma(\beta_i)} x_i^{\beta_i-1}(1-x_i)^{\alpha_i-1}, & 0 \leq x_i \leq 1, \; ; \\ 0, -\infty < x_i < 0, & 1 < x_i < \infty, \; i = 1,2,\dots,I, \end{cases} \tag{9}$$

where G - gamma-function.

Its expectation value is calculated by ratio

$$M[x_i] = \frac{\beta_i}{\alpha_i+\beta_i}, \tag{10}$$

And its variance – by the ratio

$$\sigma_i^2 = \frac{\beta_i\alpha_i}{(\alpha_i+\beta_i)^2(\alpha_i+\beta_i+1)}. \tag{11}$$

Considering (10)-(11) we can conclude that to characterize distribution (10) one must know expectation values and variances of students' competence levels. As the information available for estimating an educational process quality is depleted by the knowledge of matrix elements (1) it is expedient to use their estimations defined by this matrix as expected values and variances.

Besides expectation value of the achieved i-competence level is calculated by the ratio

$$M[\hat{x}_i] \approx \frac{\sum_{k=1}^{K} x_{ik}}{K} = \bar{x}_i \,. \qquad (12)$$

Variance σ_i^2, $(i = 1,2,\ldots,I)$of the achieved level of i-competence is determined by the ratio

$$\bar{\sigma}_i^2 = \frac{1}{K-1}\sum_{k=1}^{K}\{x_{ik} - M[\hat{x}_i]\}^2, \qquad (i = 1,2,\ldots,I), \qquad (13)$$

Where $M[\hat{x}_{ik}]$ is calculated by the formula (12).

Substituting calculated expectation values (12) or in ratio (10) and variances (13) into ratio (11) for every i(i=1,2,…,I) competence we'll obtain a system of two algebraic equations in two unknowns α_i, β_i. Having solved it for these unknowns we'll obtain

$$\beta_i = \frac{A_i - \bar{\sigma}_i^2 (A_i+1)^2}{\bar{\sigma}_i^2 (A_i+1)^3}, \qquad (14)$$

$$\alpha_i = A_i \beta_i \,, \qquad (15)$$

where $A_i = \frac{1 - M[\hat{x}_i]}{M[\hat{x}_i]}$.

Variables (14), (15) supply characterization of distribution function (9) of random quantity of competence levels achieved at the examined stage of an educational process. In view of the known function (9) the probability of the event is determined by the ratio

$$P_i(\hat{x}_i \geq x_i^{Д}) = 1 - \int_0^{x_i^{Д}} f \;\; (x_i) dx_i. \qquad (16)$$

In view of (16) quality estimation (8) of education process takes the form

$$\bar{S} = \prod_{i=1}^{I}[1 - \int_0^{x_i^{Д}} f \;\; (x_i) dx_i]. \qquad (17)$$

3. Conclusion

On the whole the obtained ratio (1)-(17) present a model which allows to estimate the quality of every stage of an educational process at any educational institution. Its application ensures an increase of decisions validity for improving the process. In view of (4) an education process is a mapping Q of available material and time resources distribution in a large number of graduates' competences and it can be applied for statistical estimation of education process quality dependence on time and material resources assigned for its realization.

References

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MODEL ZA STATISTIČKU PROCENU KVALITETA OBRAZOVNOG PROCESA

Rezime: U ovom radu ponuđen je matematički model za procenu kvaliteta obrazovnog procesa kroz realizaciju odgovarajućeg pristupa. Osnova za model je teorijsko-propabilistička formalizacija procesa formiranja kompetencija. Obrazovni proces se posmatra kao odraz vremenskih i materijalnih resursa izdvojenih u obezbeđivanju kompetencija.

Ključne reči: obrazovni proces, kompetentnost, kvalitet, verovatnoća, funkcija raspodele.